



## INTRODUCTION

Most analytical models of pulse-echo imaging derive an integration on the product of propagating pulse and the reflectors or scatterers, over the location of the pulse at some point in time (Macovski, 1983; Szabo, 2004; Prince and Links, 2006). Under a number of approximations and simplifications about attenuation and diffraction, the integration can be reduced to a convolution model (Macovski, 1983).

The problems of poor resolution and speckle can be understood as a direct result of this convolution. The spatial resolution is set by the full spatial extent of the propagating pulse in 3D which is typically many multiples of a wavelength. However, in tissue, small scatterers at the cellular level and micro-structural level such as the arterioles and capillaries will have a dimensions

zeroes of the pulse transform  $P(z)$  become the poles of the inverse filter. Generally, for a casual, right-handed system to be stable the poles of  $Z^{-1}\{P(z)\}$  must lie within the unit circle and the region of convergence

FIGURE 2. An asymmetric pulse formed by multiplying the Gaussian envelope with a geometric series is shown in (a). The zeroes of the Z-transform are retracted into the unit circle as shown in (b). This leads to a stable inverse filter.

Now all the zeroes of the transform lie within the unit circle, as seen in Figure 2b. Accordingly, the inverse filter will have poles within the unit circle and will have a bounded input/bounded output impulse response of limited duration.

In general, we have found that the formation of a stabilized pulse is not restricted to the use of a Gaussian function; rather this is illustrative of envelopes that have a sharp rise and a more gradual fall-off from the peak. We call these "asymmetric" envelopes or pulses, and these can be characterized by a number of different analytic functions. As an example, the function  $x^2 e^{-x^2/2} \text{UnitStep } x$  is selected as a stable function for the transverse beam pattern. The program Field II (Jensen, 2004) was used to simulate a focused beam pattern. The approximate Fourier transform of  $x$  is used to set the apodization function. A 5 MHz transducer with 129 active elements is simulated with half-wavelength spacing. The transverse beam pattern at the focus (60 mm depth) is shown in Figure 3. There is a good correlation between the beam pattern and the design function.

FIGURE 3. An asymmetric function (heavy line) is used as a design for a focal transverse beam pattern (squares) at 5 MHz. Vertical axis: magnitude of the focal transverse beam pattern (arbitrary units)

## DISCUSSION AND CONCLUSIONS

An inverse filter approach has been derived using

Szabo, T. L. 2004. "Wave scattering and imaging," *Diagnostic Ultrasound Imaging : Inside Out* (Elsevier Academic Press, Amsterdam ; Boston), pp. 213-242.

Taxt, T., and Frolova, G. V. 1999. "Noise robust one-dimensional blind deconvolution of medical ultrasound images," *IEEE Trans Ultrason Ferroelectr Freq Control* 46, 291-299.

Tuthill, T. A., Sperry, R. H., and Parker, K. J. 1968. "Deviations from Rayleigh statistics in ultrasonic speckle," *Ultrason Imaging* 10, 81-89.

Wagner, R. F., Smith, S. W., Sandrik, J. a966E( in ultrass3u2driA., Spe81-)5.1ccl5.4(,0e2.4(s)8.8( in ultraB1ti)5.4(cs)8.8n n ultra2Te